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Topic: Rules of Differentiation

Product Rule of Differentiation

According to the product rule of derivatives, if the function f(x) is the product of two functions u(x) and v(x), then the derivative of the function is given by:

If $f(x) = u(x) \times v(x)$, then:

$$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$$

Example: Find the derivative of $x^2(x+3)$.

Solution: As per the product rule of derivative, we know;

$f'(x) = u'(x) \times v(x) + u(x) \times v'(x)$

Here,

$u(x) = x^2$ and $v(x) = x+3$

Therefore, on differentiating the given function, we get;

$f'(x) = d/dx[x^2(x+3)]$

$f'(x) = d/dx(x^2)(x+3)+x^2 d/dx(x+3)$

$f'(x) = 2x(x+3)+x^2(1)$

$f'(x) = 2x^2+6x+x^2$

$f'(x) = 3x^2+6x$

$f'(x) = 3x(x+2)$

Quotient Rule of Differentiation

If f(x) is a function, which is equal to ratio of two functions u(x) and v(x) such that;

$f(x) = u(x)/v(x)$

Then, as per the quotient rule, the derivative of f(x) is given by;

$$\mathbf{f'(x) = \frac{u'(x) \times v(x) - u(x) \times v'(x)}{(v(x))^2}}$$

Example: Differentiate $f(x)=(x+2)^3/\sqrt{x}$

Solution: Given,

$f(x)=(x+2)^3/\sqrt{x}$

$= (x+2)(x^2+4x+4)/\sqrt{x}$

$= [x^3+6x^2+12x+8]/x^{1/2}$

$= x^{-1/2}(x^3+6x^2+12x+8)$

$= x^{5/2}+6x^{3/2}+12x^{1/2}+8x^{-½}$

Now, differentiating the given equation, we get;

$f'(x) = 5/2x^{3/2} + 6(3/2x^{1/2})+12(1/2x^{-1/2})+8(-1/2x^{-3/2})$

$= 5/2x^{3/2} + 9x^{1/2} + 6x^{-½} - 4x^{-3/2}$

Chain Rule of Differentiation

If a function y = f(x) = g(u) and if u = h(x), then the chain rule for differentiation is defined as;

dy/dx = (dy/du) × (du/dx)

This rule is majorly used in the method of substitution where we can perform differentiation of composite functions.

Let's have a look at the examples given below for better understanding of the chain rule differentiation of functions.

Example 1:

Differentiate $f(x) = (x^4 – 1)^{50}$

Solution:

Given,

$f(x) = (x^4 – 1)^{50}$

Let $g(x) = x^4 – 1$ and n = 50

$u(t) = t^{50}$

Thus, $t = g(x) = x^4 – 1$

f(x) = u(g(x))

According to chain rule,

df/dx = (du/dt) × (dt/dx)

Here,

du/dt = d/dt (t50) = $50t^{49}$

dt/dx = d/dx g(x)

$= d/dx\ (x^4 – 1)$

$= 4x^3$

Thus, $df/dx = 50t^{49} \times (4x^3)$

$= 50(x^4 – 1)^{49} \times (4x^3)$

$= 200\ x^3(x^4 – 1)^{49}$

Example 2:

Find the derivative of $f(x) = e^{\sin(2x)}$

Solution:

Given,

$f(x) = e^{\sin(2x)}$

Let $t = g(x) = \sin 2x$ and $u(t) = e^t$

According to chain rule,

$df/dx = (du/dt) \times (dt/dx)$

Here,

$du/dt = d/dt\ (e^t) = e^t$

$dt/dx = d/dx\ g(x)$

$= d/dx\ (\sin 2x)$

$= 2 \cos 2x$

Thus, $df/dx = e^t \times 2 \cos 2x$

$= e^{\sin(2x)} \times 2 \cos 2x$

$= 2 \cos(2x)\ e^{\sin(2x)}$